Connecting Models with Data for Coupled Online–Offline Social Systems

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- **Fact:**

  [Online-offline spillovers] Social media is transforming how, when, and where conflicts occur.

  About 68.7% of the global population uses social media (Digital 2026 Global Overview Report by DataReportal).

  New online connections —> 7% increase in physical activity (Althoff *et al.* 2017).

  Online communication and on-site protests exhibit bidirectional influence, as illustrated by the worldwide Occupy movement in 2011 (Bastos *et al.* 2015).

- **Goal:** Understand how online and offline activities influence one another.

Two approaches: **Model-Driven** & **Data-Driven**

Modeling Coupled Online—Offline Dynamics

U: Uninterested

E: Engaged

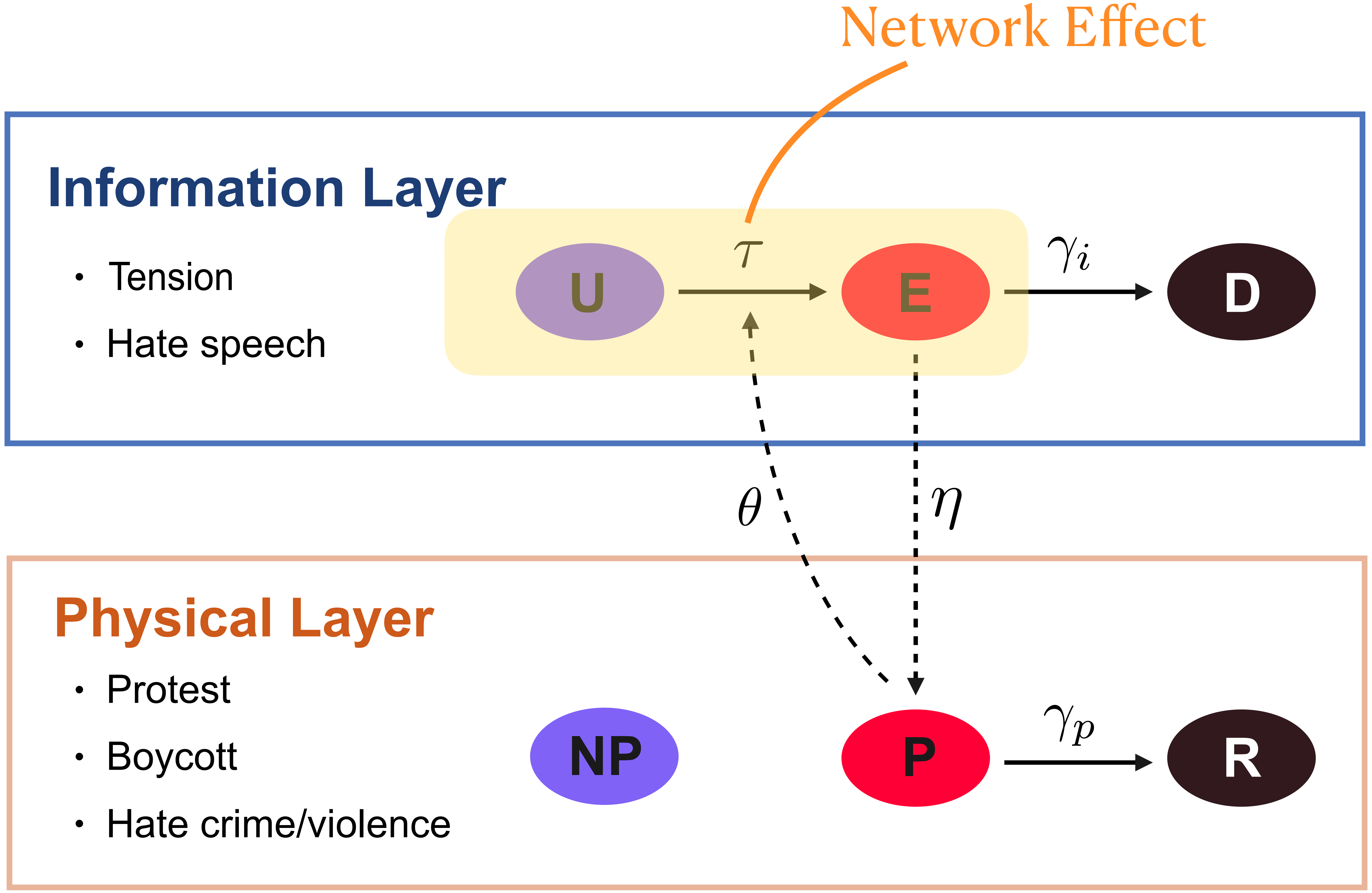
D: Disengaged

τ : transmission rate
γ_i : recovery rate
η : transmission rate
γ_p : recovery rate
θ : self-excitement

NP: Non-participating

P: Participating

R: Recovered from Participation



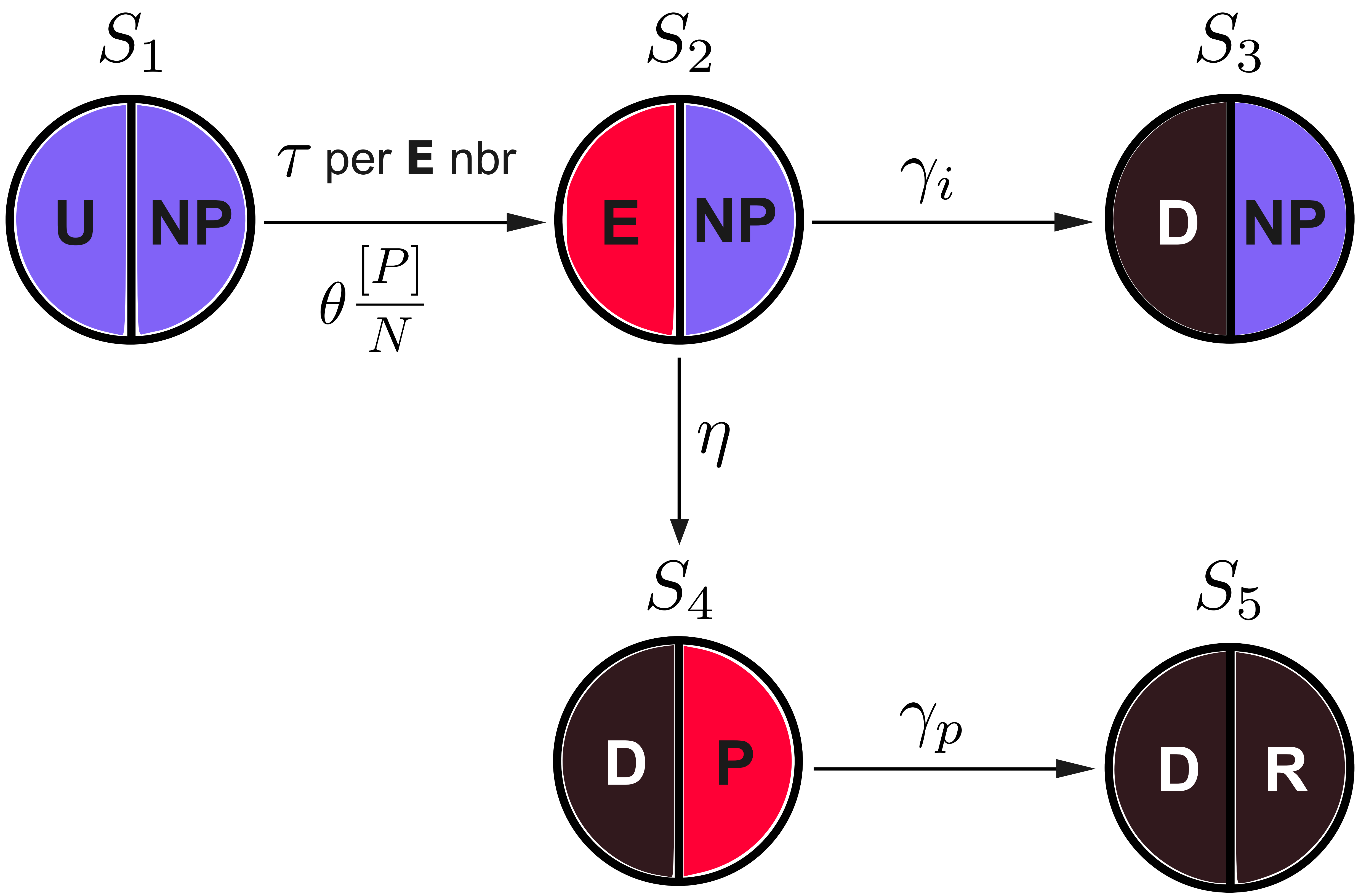
- Multi-layer network compartmental model incorporating the coupled effects of online engagement and offline protests

[Tian, Brantingham & Rodríduez, J. Complex Netw. (2026)]

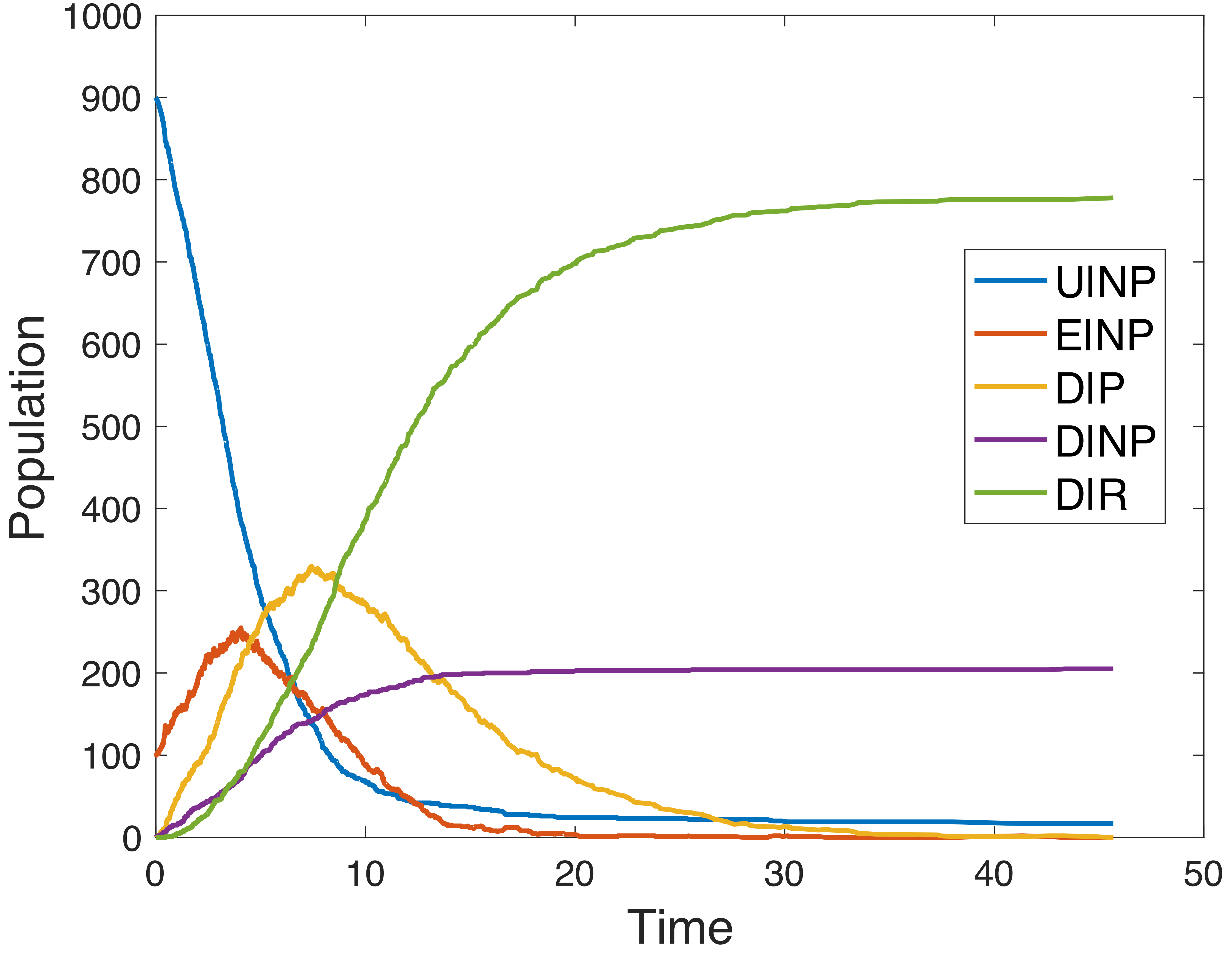
- Continuous-Time Markov chain

N: Total # of nodes

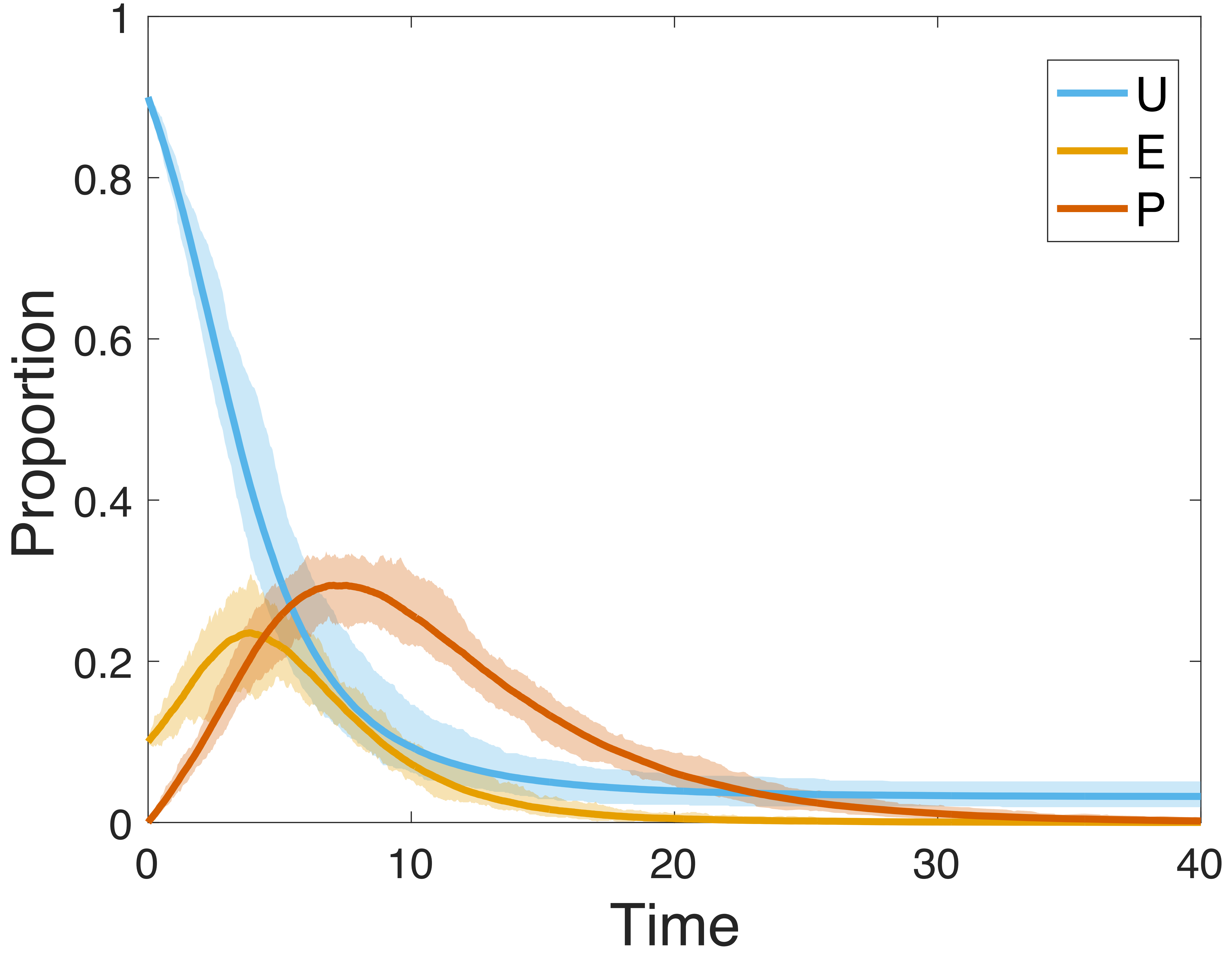
[X]: Expected # of individuals in state X



Single simulation:



Mean aggregated trajectories from multiple simulations (in ratios):



- Markovian Stochastic Process —> ODE Model

[XY]: Expected # of Y neighbors of all Xs

Online

$$\begin{cases} \dfrac{d[U](t)}{dt} = -\tau[UE] - \dfrac{\theta}{N}[U][P] \\ \dfrac{d[E](t)}{dt} = \tau[UE] + \dfrac{\theta}{N}[U][P] - (\eta + \gamma_i)[E] \\ \dfrac{d[D](t)}{dt} = (\eta + \gamma_i)[E] \end{cases}$$

Offline

$$\begin{cases} \dfrac{d[P](t)}{dt} = \eta[E] - \gamma_p[P] \\ \dfrac{d[R](t)}{dt} = \gamma_p[P] \end{cases}$$

Need to approximate the [UE] term to close the system

-> **mean-field approximation** that depends on network structures

- Mean-field models can be derived under
  - Different **assumptions on network structures** (fully-connected? Homogeneous/heterogeneous degree?...)
  - & Different **levels of approximation accuracy** (node-level, pairwise, triplet, with the truncation chosen to balance accuracy and tractability)

For Example:

- If assuming **Fully-Connected Network**: An average engaged individual has approximately $[U]/N$ uninterested neighbors —> approximate [UE] by $\frac{[U]}{N}[E]$
- If assuming **Homogeneous Network** (k-regular graph):
  - **Single-Level (Mean-Field) Approximation**: Pair counts are approximated using singleton counts.

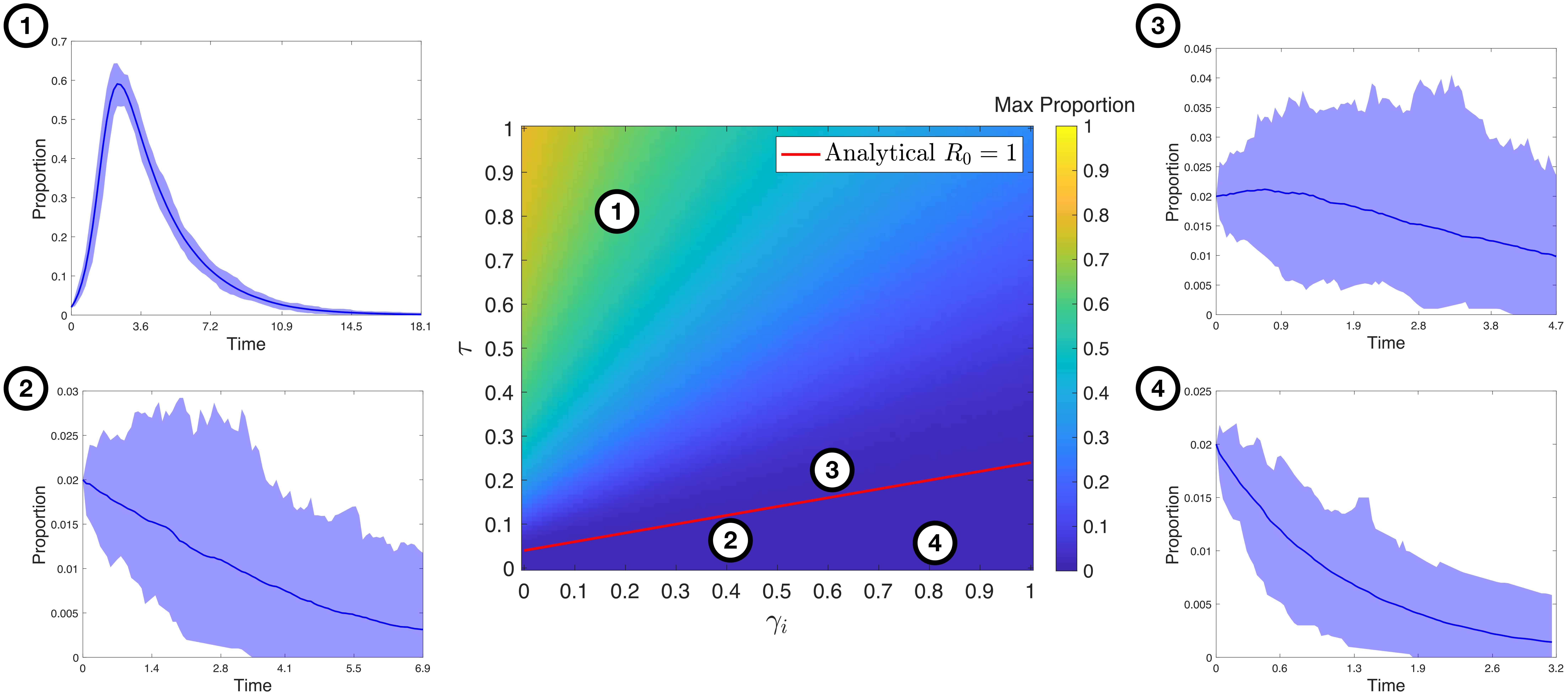
    Assume that engaged individuals are distributed randomly, so $[UE] \approx \frac{k}{N}[U][E]$
  - OR **Pairwise Approximation**: We explicitly track the evolution of pair counts, which depends on triplet counts, and close the system by approximating triplets in terms of pairs.

    Key idea: The probability of choosing X&Z to be neighbors of Y is approximately $\frac{[XY][YZ]}{k^2[Y]}$. There are $k(k-1)$ ways to choose 2 neighbors.
- If assuming **Heterogeneous Network**: Similar approach, but nodes are differentiated by degree, and state counts are tracked within each degree class

[Kiss, Miller & Simon, Mathematics of Epidemics on Networks: From Exact to Approximate Models, Springer (2017)]

Application: Reproductive Number R_0

The average number of newly engaged individuals motivated by a single engaged person in the pre-engaged online population



For Online Engagement E

* R_0 calculated from the stochastic process

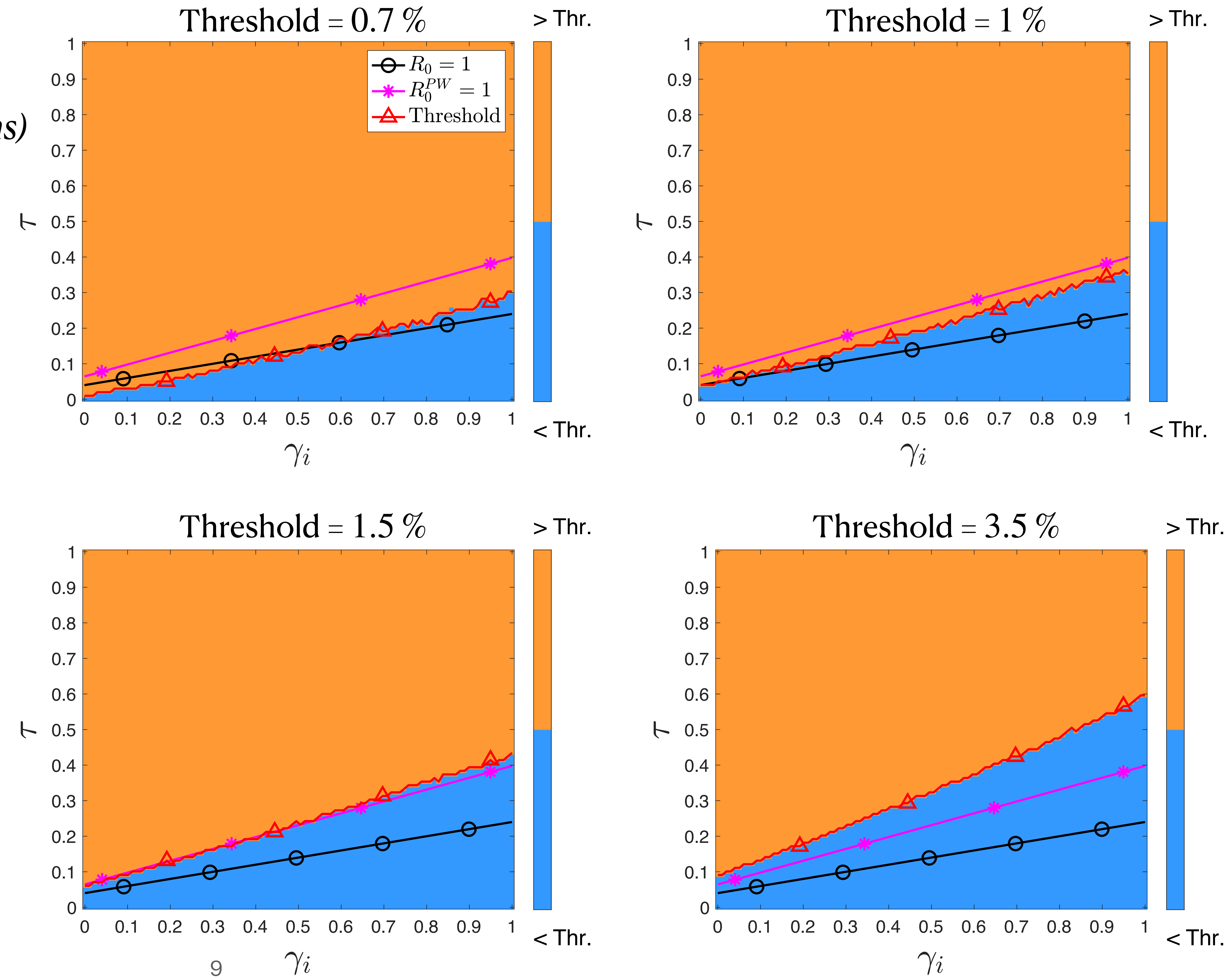
Categorization of Offline Outbursts & Comparison

* R_0^{PW} from the pairwise approximation model
(more accurate, as it incorporates inter-layer interactions)

- Categorization of offline outbursts based on threshold for the max proportion

- Protests exceeding 3.5% active participation at peak activity have historically tended to succeed (Chenoweth, 2020).

- The $R_0 = 1$ threshold is more conservative than $R_0^{PW} = 1$; however, both appear overly conservative relative to empirical observations.



- **Goal:** Understand how online and offline activities influence one another.

Two approaches: **Model-Driven** & **Data-Driven**

Two Attempts:

⇨ System Identification by Weak Form SINDy

⇨ Model Inference from Multi-Agent LLM Dynamics

System Identification by Weak Form SINDy

SINDy

Sparse Identification of Nonlinear Dynamics

Method for learning governing equations $\dot{\mathbf{x}}(t) = \mathbf{F}(\mathbf{x}(t))$ from observed data observed at timepoints $t_1, t_2, \ldots, t_M$

$$\mathbf{X} = [\mathbf{x}(t_1) + \epsilon_1 \quad \mathbf{x}(t_2) + \epsilon_2 \quad \ldots \quad \mathbf{x}(t_M) + \epsilon_M]^T$$

using sparse regression on

$$\dot{\mathbf{X}} = \Theta(\mathbf{X})\,\mathbf{w}$$

where $\Theta(\mathbf{X}) = [f_1(\mathbf{x}) \quad f_2(\mathbf{x}) \ \cdots\ ]$ are candidate functions in the chosen library.

Solving a sparse $\mathbf{w}^*$ by minimizing $\|\dot{\mathbf{X}} - \Theta(\mathbf{X})\,\mathbf{w}\|_2$ —> Discover a model with a few terms

[Brunton, Proctor & Kutz, Proc. Natl. Acad. Sci (2016)]

WSINDy

Weak form SINDy

To avoid approximation for derivatives sensitive to noise, we can solve the generalized least squares problem $G\mathbf{w} = b$ in weak form where $G \in \mathbb{R}^{L\times J}$ and $b \in \mathbb{R}^{L\times d}$ are

$$G_{l,j} = \langle \phi_l, f_j(\mathbf{X})\rangle \quad \text{and} \quad b_{l,i} = -\langle \phi_l', \mathbf{X}_i\rangle.$$

$\{\phi_l\}_{l=1,\ldots,L}$ is a family of smooth compactly supported test functions.

When using multiple trajectory data, we solve:

$$\begin{bmatrix} G^1 \\ G^2 \\ \vdots \\ G^K \end{bmatrix} \mathbf{w} = \begin{bmatrix} b^1 \\ b^2 \\ \vdots \\ b^K \end{bmatrix}$$

where K is the number of trajectories included in the learning.

* GitHub page for WSINDy group: https://github.com/MathBioCU

[Messenger & Bortz, Multiscale Model. Simul. (2021)]

Results

* On Erdős–Rényi with $N = 1000$, $k = 5$

May suggest a better approximation model !

(particularly on networks with low average degree when mean-field performs poorly)

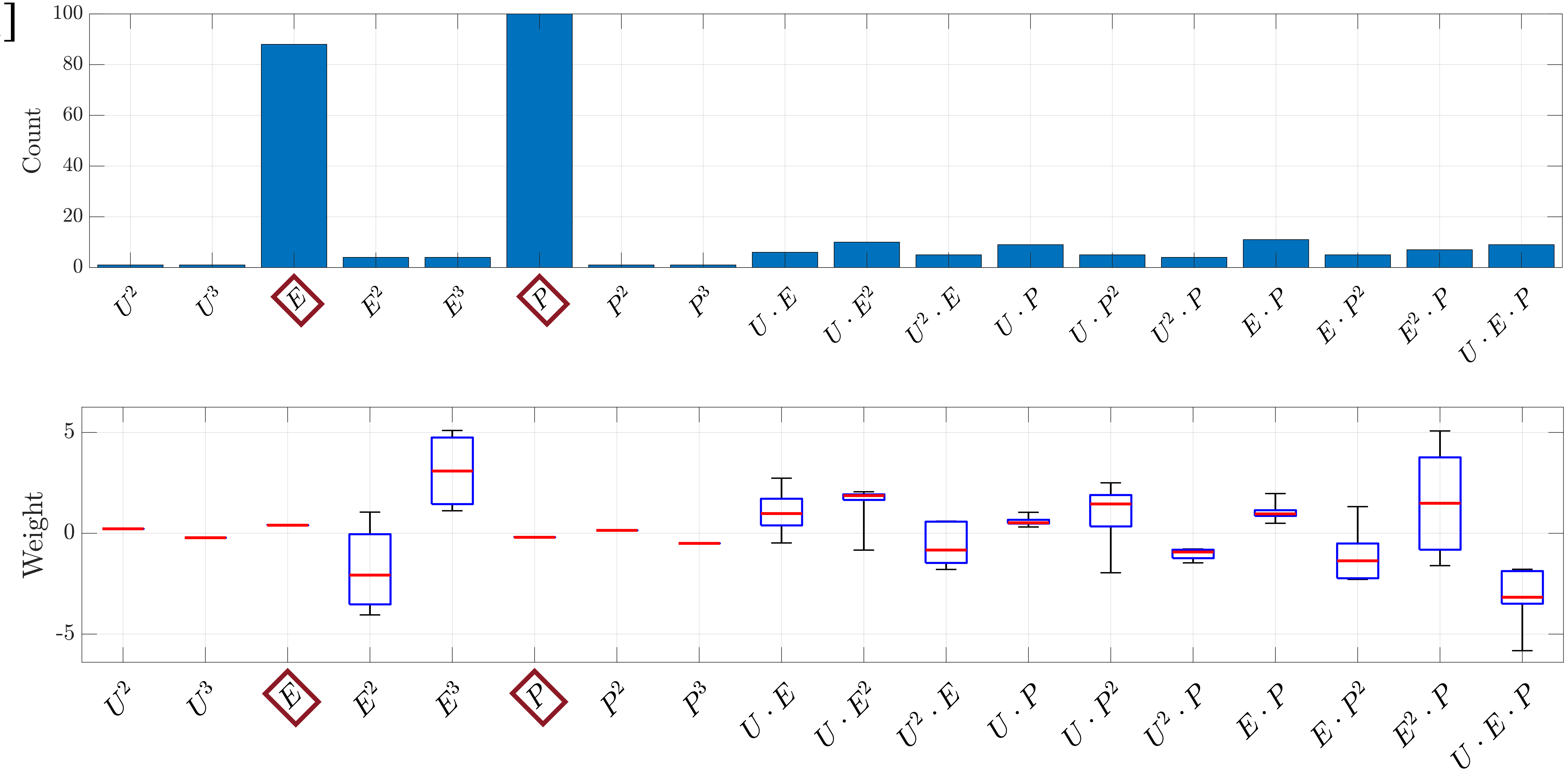
* GitHub: https://github.com/Moyi-Tian/WSINDy-NetworkDynamics

[Tian, Messenger, Dukic, Rodríduez & Bortz, arXiv (2026)]

A Bit More on the Learned Models

Terms & Weight learned across 100 stochastic data

[P Equation]

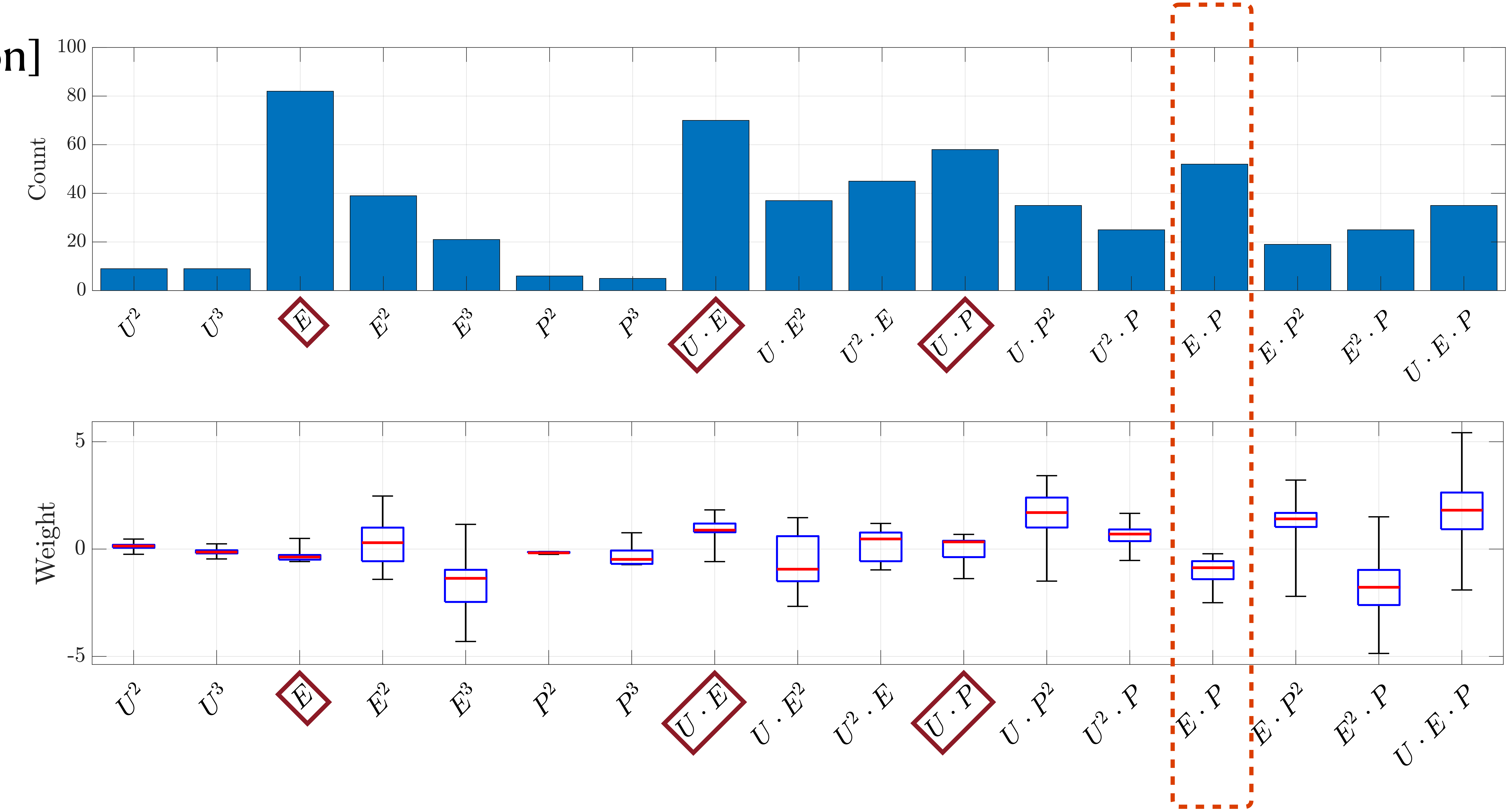


These are right on!

* Terms boxed in red are present in the mean-field

Highest-frequency non-mean-field candidate term from WSINDy

[E Equation]



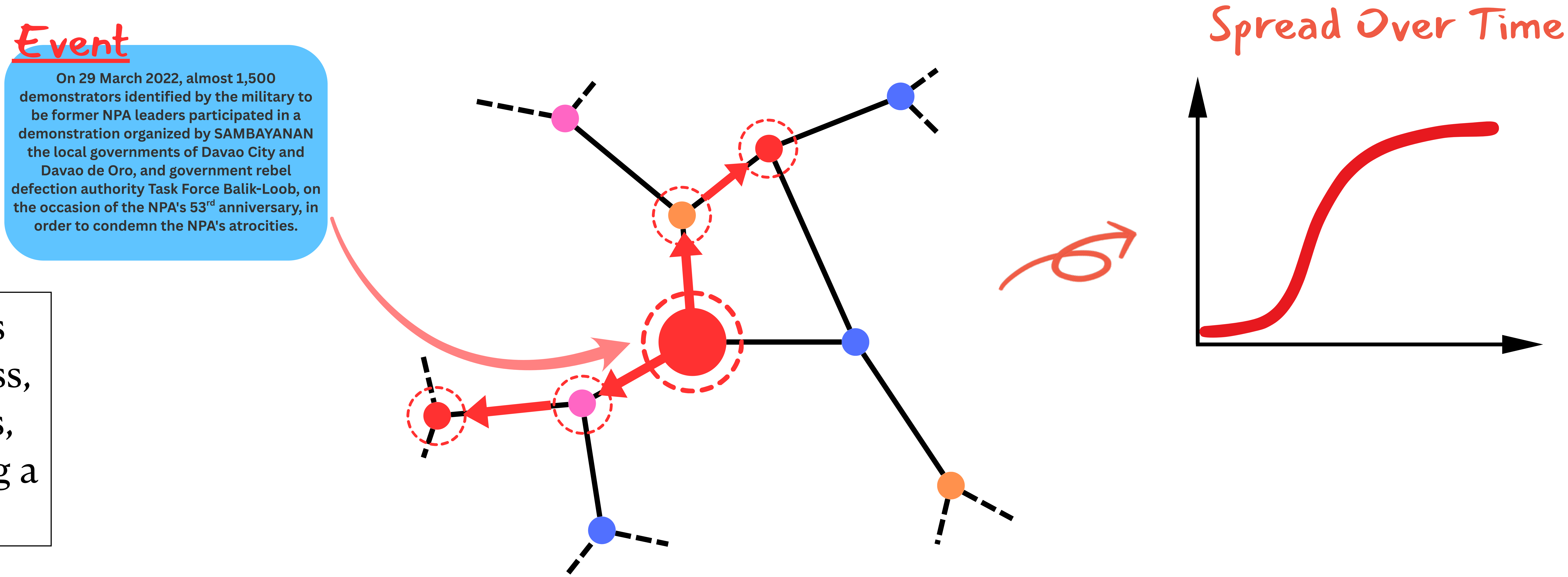
* Terms boxed in red are present in the mean-field

Model Inference from Multi-Agent LLM Dynamics

- Motivation:

More than half of the Internet traffic in 2023 was generated by bots on social media (Ng & Carley, Scientific Reports 2025).

➡ Bots are highly active online, occupying much of the traffic and exerting significant influence.

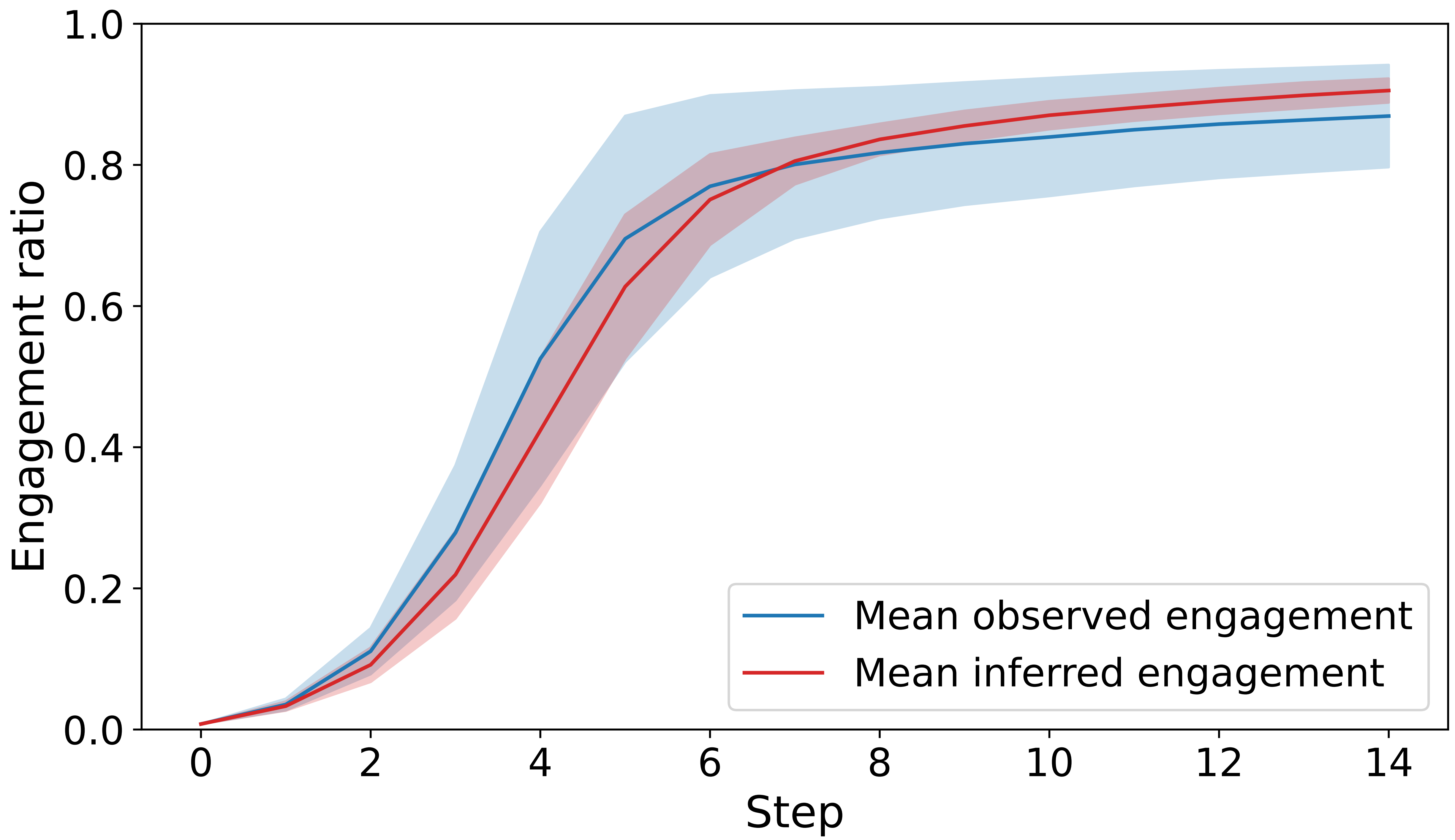


Assign agents *Big Five* traits (openness, conscientiousness, extraversion, agreeableness, and neuroticism), each taking a binary value of low or high

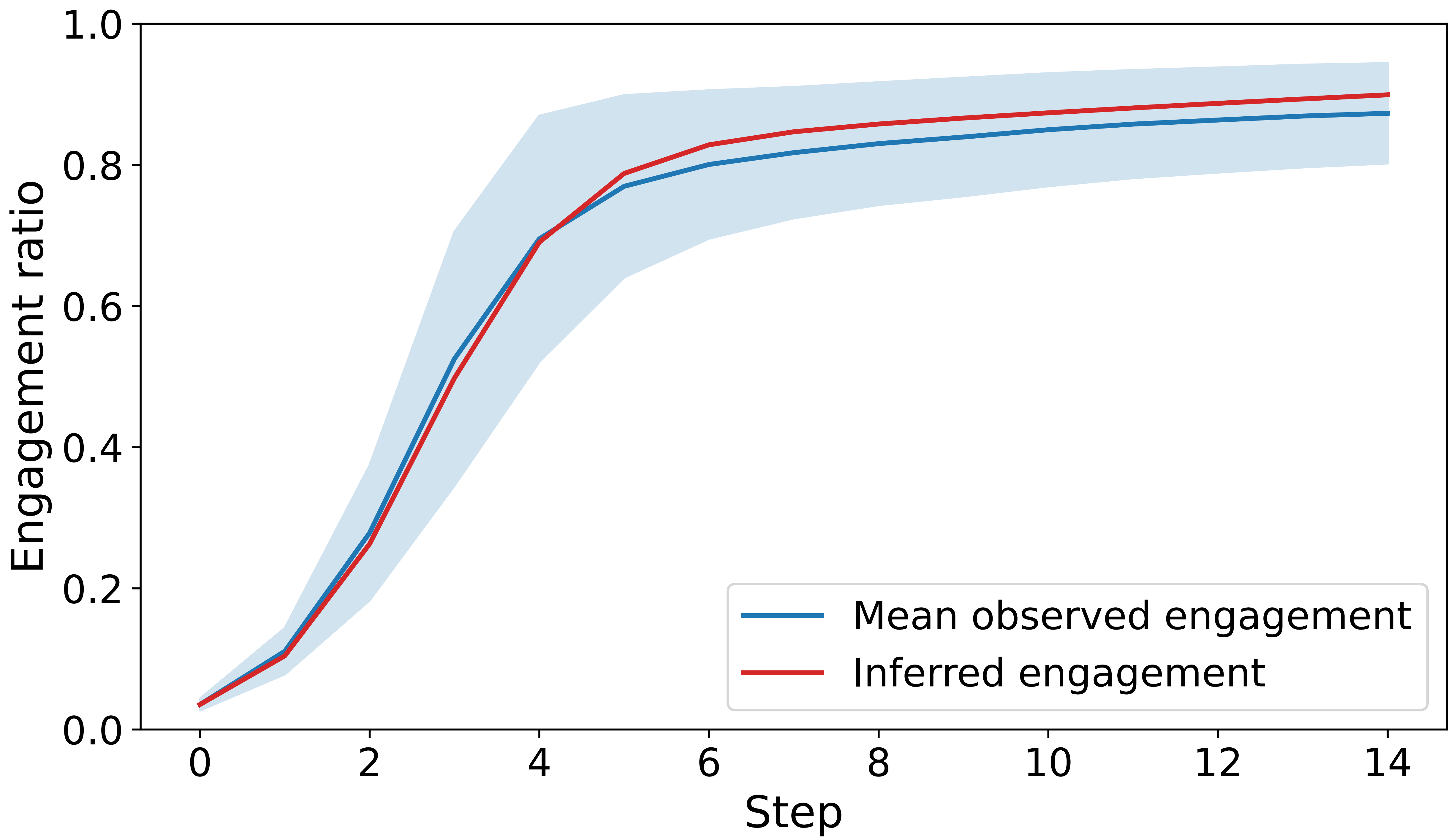
*Events taken from ACLED Philippines data (2021-02-24 ~ 2024-02-27).

[Tian, Mohler, Brantingham & Rodríduez, arXiv (2026)]

Model Fitting from Data



Logistic Model



2-group Mean-Field Model

Simpler and Better !

* GitHub: https://github.com/Moyi-Tian/Learning-LLM-Dynamics

Summary

- We developed a two-layer model for online–offline dynamics, derived corresponding mean-field approximations, and analyzed outburst behavior using the reproductive number.
- From a data-driven perspective, we investigated the learning of effective models in two complementary projects, representing initial steps toward bridging model-driven and data-driven approaches for online–offline social systems on networks.

Takeaway: ODE approximation models can effectively capture dynamics on complex networks and provide a useful framework for subsequent analysis.

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3. Moyi Tian, George Mohler, P. Jeffrey Brantingham, and Nancy Rodríguez. Learning dynamics from online-offline systems of LLM agents. arXiv:2602.23437, 2026.

[See paper]

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