Modeling Coupled Online and Offline Dynamics of Protesting Activity on Networks

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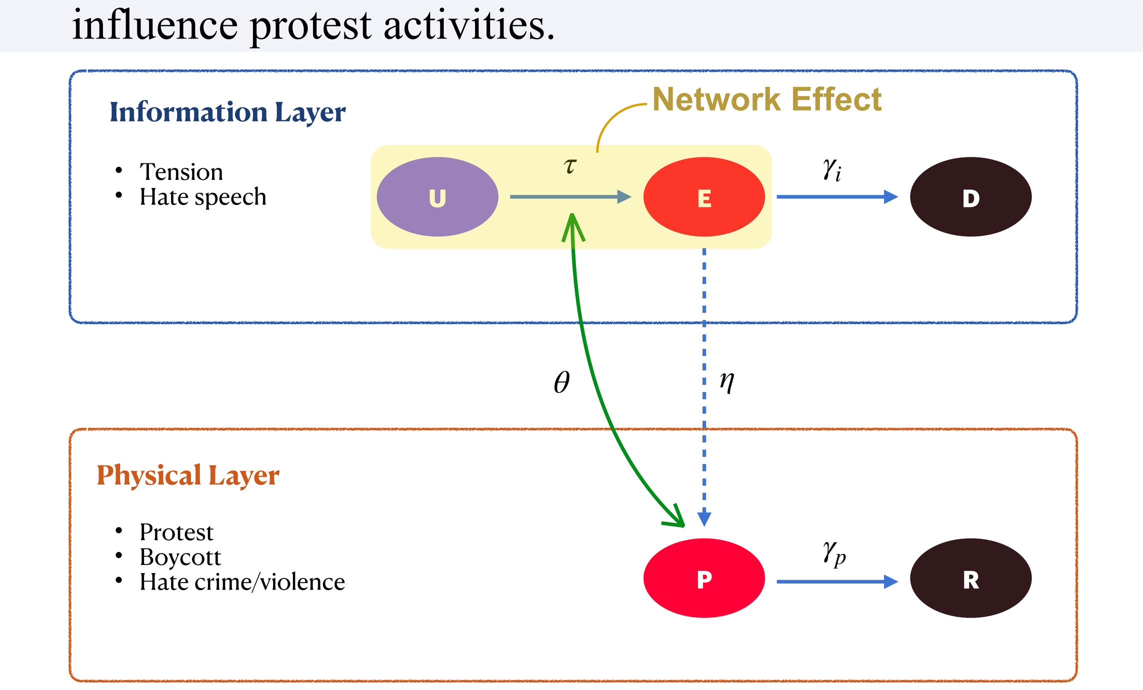
Introduction

- Social media is reshaping how, when, and where conflicts emerge.
- Over 60% of the global population uses social media (Digital 2024 Global Overview Report, DataReportal).
- **Goal:** We propose a mathematical framework to model and analyze how coupled online-offline dynamics influence protest activities.



Online States

- **U:** Uninterested
- **E:** Engaged
- **D:** Disengaged

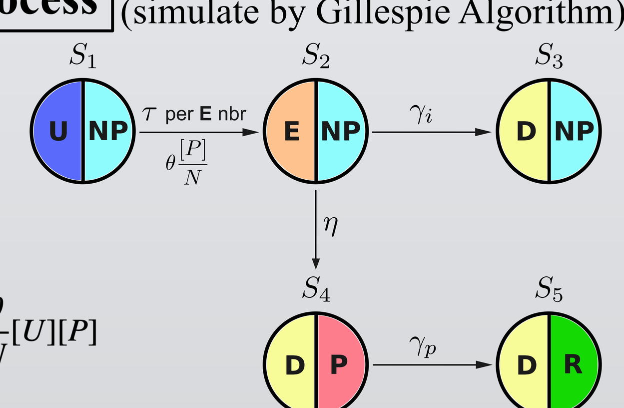
Offline States

- **NP:** Non-protesting (Not shown; default state)
- **P:** Protesting
- **R:** Recovered from Protesting

τ : Online transmission rate
γ_i : Online recovery rate
η : Offline transmission rate
γ_p : Offline recovery rate
θ : Self-excitement

Mean-Field Approximation

Markovian Stochastic Process (simulate by Gillespie Algorithm)



ODE Model

Online

$$\begin{cases} \dfrac{d[U](t)}{dt} = -\tau[UE] - \dfrac{\theta}{N}[U][P] \\ \dfrac{d[E](t)}{dt} = \tau[UE] + \dfrac{\theta}{N}[U][P] - (\eta+\gamma_i)[E] \\ \dfrac{d[D](t)}{dt} = (\eta+\gamma_i)[E] \end{cases}$$

Offline

$$\begin{cases} \dfrac{d[P](t)}{dt} = \eta[E] - \gamma_p[P] \\ \dfrac{d[R](t)}{dt} = \gamma_p[P] \end{cases}$$

Notations

- $[X]$: Expected # of individuals in state X
- $[XY]$: Expected # of Y neighbors of all Xs
- $[XYZ]$: Expected # of triples with Y having both X&Z neighbors
- $[X_n]$: Expected # of individuals in state X with degree n
- $[n]$: Expected # of individuals with degree n

- The continuum ODE model describes the expected number of individuals in each state during the stochastic evolution, which is the quantity of interest.
- **Challenge:** The term $[UE]$ depends on the exact network structure, whose time-dependent values are unknown.
- **Approach:** We approximate $[UE]$ using **mean-field methods** to obtain closed ODE systems, enabling further analysis.

Single-Level Approximation: Assume random distribution of individuals in each state across the network

Pairwise Approximation:

N = Total number of nodes
k = Average degree
K = Maximum degree

We consider edge evolution:

$$\begin{cases} \dfrac{d[UE](t)}{dt} = -(\gamma_i+\eta+\tau)[UE] + \tau([UUE]-[EUE]) + \dfrac{\theta[P]}{N}[UU] - \dfrac{\theta[P]}{N}[UE] \\ \dfrac{d[UU](t)}{dt} = -\tau([UUE]+[EUU]) - \dfrac{2\theta[P]}{N}[UU] \end{cases}$$

- We now approximate the triplets $[UUE]$ and $[EUE]$

Our Models

On Homogeneous Networks

* Consider k-regular networks (Fully-connected network is a special case)

Single-Level Approximation

$$\begin{cases} \frac{d[U](t)}{dt} = -\tau\frac{k}{N}[U][E] - \frac{\theta}{N}[U][P] \\ \frac{d[E](t)}{dt} = \tau\frac{k}{N}[U][E] + \frac{\theta}{N}[U][P] - (\eta+\gamma_i)[E] \\ \frac{d[D](t)}{dt} = (\eta+\gamma_i)[E] \\ \frac{d[P](t)}{dt} = \eta[E] - \gamma_p[P] \\ \frac{d[R](t)}{dt} = \gamma_p[P] \end{cases}$$

Pairwise Approximation

$$\begin{cases} \frac{d[U](t)}{dt} = -\tau[UE] - \frac{\theta}{N}[U][P] \\ \frac{d[E](t)}{dt} = \tau[UE] + \frac{\theta}{N}[U][P] - (\eta+\gamma_i)[E] \\ \frac{d[D](t)}{dt} = (\eta+\gamma_i)[E] \\ \frac{d[UE](t)}{dt} = -(\gamma_i+\eta+\tau+\frac{\theta[P]}{N})[UE] + \frac{\theta[P]}{N}[UU] + \frac{\tau(k-1)}{k[U]}\left([UU][UE]-[EU]^2\right) \\ \frac{d[UU](t)}{dt} = -2\tau\left(\frac{k-1}{k}\frac{[UU][UE]}{[U]}\right) - \frac{2\theta[P]}{N}[UU] \\ \frac{d[P](t)}{dt} = \eta[E] - \gamma_p[P] \\ \frac{d[R](t)}{dt} = \gamma_p[P] \end{cases}$$

Assume that edges connecting nodes in any pair of states are uniformly distributed

On Heterogeneous Networks

* Group the dynamics by node degree, $n = 1, \ldots, K$

Single-Level Approximation

$$\begin{cases} \frac{d[U_n]}{dt} = -\tau n[U_n]\pi_E - \frac{\theta}{N}[U_n][P] \\ \frac{d[E_n]}{dt} = \tau n[U_n]\pi_E + \frac{\theta}{N}[U_n][P] - (\eta+\gamma_i)[E_n] \\ \frac{d[D_n]}{dt} = (\eta+\gamma_i)[E_n] \\ \frac{d[P](t)}{dt} = \eta[E] - \gamma_p[P] \\ \frac{d[R](t)}{dt} = \gamma_p[P] \end{cases}$$

where $\pi_E = \frac{\sum_{j=1}^K j[E_j]}{\sum_{j=1}^K j[j]}$

Approximate $[U_nE_m]$ for all degree pairs (n, m) and additionally assume random edge connections between nodes of any two degrees

Pairwise Approximation

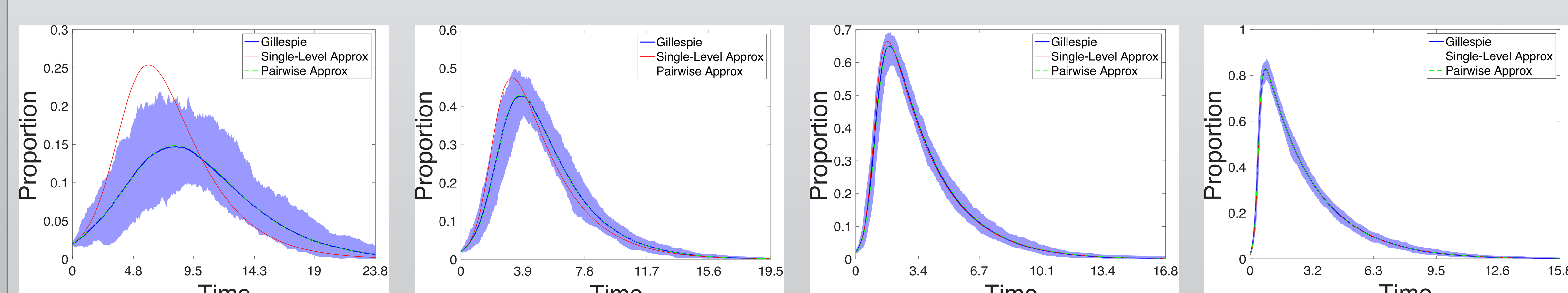
$$\begin{cases} \frac{d[U_n]}{dt} = -\tau n[U_n]\frac{[UE]}{[UX]} - \frac{\theta}{N}[U_n][P] \\ \frac{d[E_n]}{dt} = \tau n[U_n]\frac{[UE]}{[UX]} + \frac{\theta}{N}[U_n][P] - (\eta+\gamma_i)[E_n] \\ \frac{d[D_n]}{dt} = (\eta+\gamma_i)[E_n] \\ \frac{d[UE]}{dt} = \tau Q_c[UE]([UU]-[UE]) - (\tau+\eta+\gamma_i+\frac{\theta}{N}[P])[UE] + \frac{\theta}{N}[P][UU] \\ \frac{d[UU]}{dt} = -2\tau Q_c[UU][UE] - \frac{2\theta}{N}[P][UU] \\ \frac{d[P](t)}{dt} = \eta[E] - \gamma_p[P] \\ \frac{d[R](t)}{dt} = \gamma_p[P] \end{cases}$$

where $[UX] = \sum_{l=1}^K l[U_l]$ and $Q_c = \frac{1}{[UX]^2}\sum_{k=1}^K (k-1)k[U_k]$

Assume neighbors of all uninterested individuals are interchangeable when approximating triplet terms across degrees

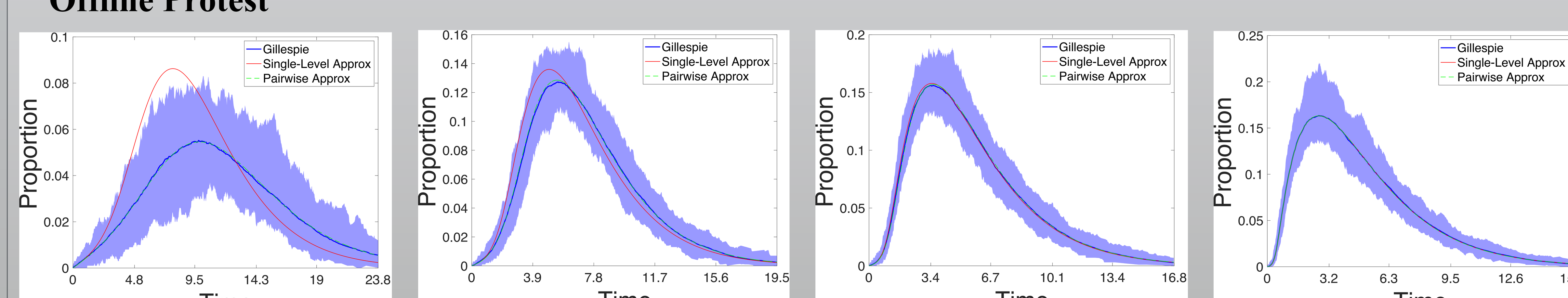
Simulations

Online Engagement



$k = 5$ $k = 10$ $k = 20$ $k = 50$

Offline Protest

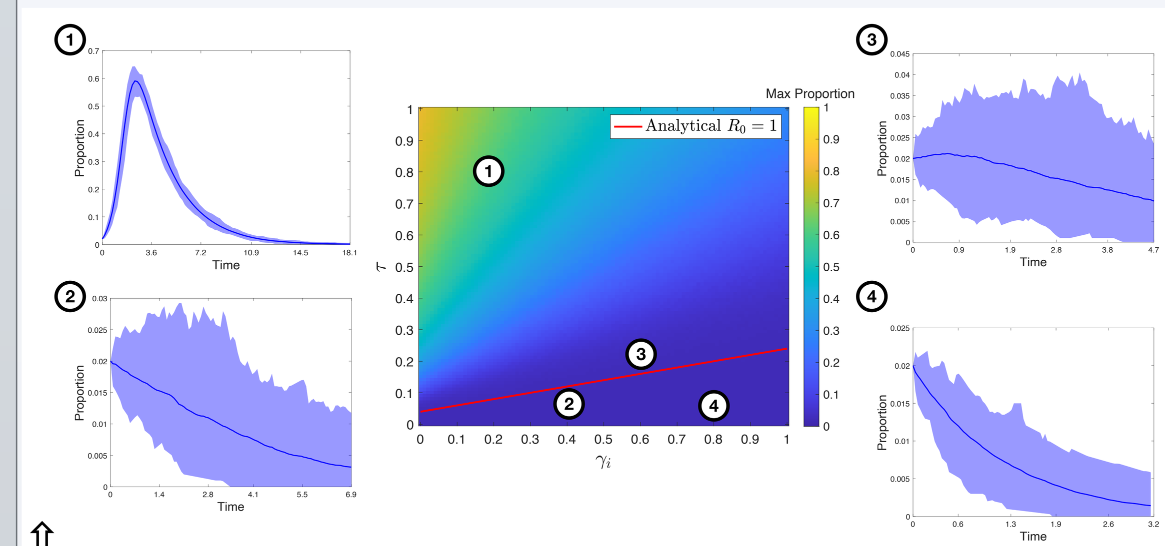


$k = 5$ $k = 10$ $k = 20$ $k = 50$

Figure. Dynamics of online engagement and offline protest on Erdős–Rényi networks with varying average degree k, while fixing $N = 1000$. The pairwise approximation is more accurate than the single-level approximation. Additionally, as k increases, the accuracy of the single-level approximation improves, since the random mixing assumption underlying the approximation becomes more valid.

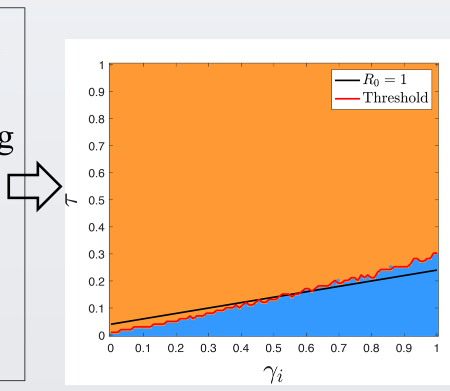
Outburst Analysis

The models can be leveraged to investigate how system parameters influence the online and offline outbursts.



We analyze online engagement outburst sizes across the γ_i-τ plane and derive the reproductive number R_0, using $R_0 = 1$ as a classification threshold. Example dynamics from different parameter regions are shown.

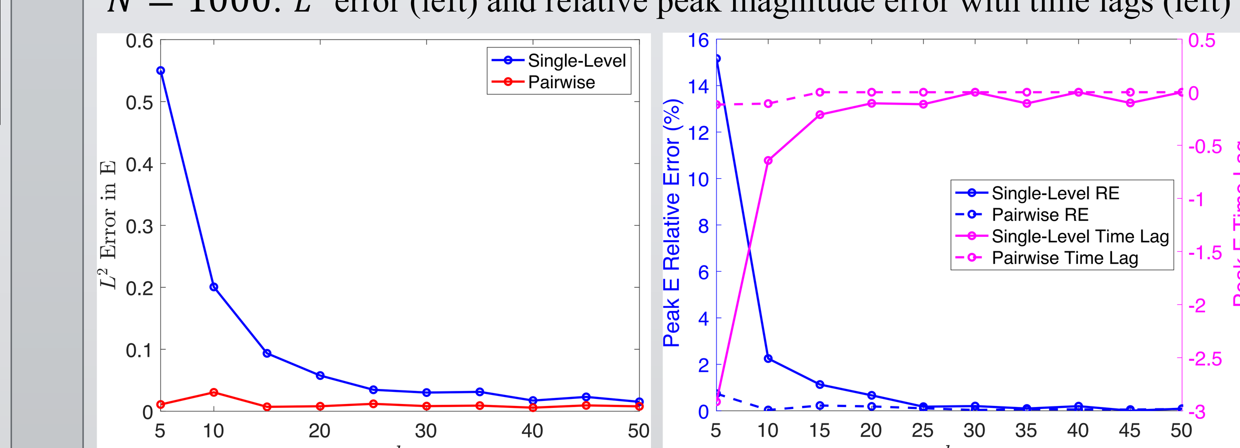
To interpret the empirical meaning of R_0, we define a classification threshold (Thr. = 0.007 in the plot) for offline protest outbursts and compare it to the $R_0 = 1$ threshold. According to political scientist Erica Chenoweth*, campaigns with at least 3.5% active participation have historically never failed, suggesting this may be a critical threshold. In contrast, the analytical $R_0 = 1$ is more conservative.



* Woods, D. (2019, June 25). *The magic number behind protests*. NPR. https://www.npr.org/sections/money/2019/06/25/735536434/the-magic-number-behind-protests

Approximation Errors

Approximation errors for online engagement on k-regular networks with $N = 1000$: L^2 error (left) and relative peak magnitude error with time lags (left)



The pairwise approximation is generally more accurate than the single-level. As network density increases, single-level errors drop significantly, in contrast to pairwise errors. Both approximations predict earlier peak times, with the single-level showing a greater time lead.

Discussion

Takeaways:

- Network structure impacts the accuracy of mean-field approximations.
- Low-density networks require more complex models to achieve the accuracy of simpler ones on high-density networks.
- Model selection should reflect network structure and desired approximation accuracy.

Future Directions:

- Develop mean-field models that incorporate clustering and explore alternative approximation methods.
- Use continuum models to integrate with data for tasks like parameter estimation and system identification.

References

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2.IZ Kiss , JC Miller , PL Simon. Mathematics of Epidemics on Networks: From Exact to Approximate Models. Springer, 2017